Title: Heart Rates as a Sinusoidal Curve

Link to Outcomes:

- **Problem Solving** Students will perform harmonic analysis (finding the equation, given the curve).
- **Communication** Students will discuss mathematical concepts with other students and make predictions.
- **Reasoning Skills** Students will gather evidence, make conjectures, and test conjections.
- **Connections** Students will observe connections between trigonometry (sinusoidal curves) and biology (heartbeats).
- **Trigonometry** Students will use the properties of a cosine curve to determine the variables in y=Acos(B(x-C)+D)
- **Technology** Students will use the Calculator Based Laboratory (CBL), the heart monitor, and TI-82 Graphics Calculator (TI-82) to create a sinusoidal curve that corresponds to their heartbeats.
- **Cooperation** Students will demonstrate the ability to investigate bio-mathematical concepts while working in small groups.

Brief Overview:

Sinusoidal functions can be used to represent repetitive behavior. In this lesson, students will record their heartbeats using a CBL and a heart monitor; based on this data, the students will plot these values using the TI-82. They will then estimate a "best fit" cosine curve. This equation can be matched with a TI-82 generated equation.

Grade/Level:

Grades 11-12: Trig/Pre-calculus

Duration:

This lesson is expected to take one class period (approximately 45 minutes) but it could be extended to another period, if one so chooses.

Prerequisite Knowledge:

Students must understand the relationships between the coefficients of y=Acos(B(x-C)+D) and its graph.

Objectives:

Students will

- determine the cosine equation that represents the graph.
- recognize the relationship between trigonometry and biology.
- determine which coefficient is effected by exercise (optional).
- experiment to determine if the heart rate of male versus female might affect the coefficients (optional).

Materials/Resources/Printed Materials:

- Computer Programs: Heart: Exploring Physics w/CBL (TI-82)
  ACOS: John Diehl, Hinsdale Central High School
  ALLSELCT and SORTLIST: Phillip Sanderson
- Heart Monitor, CBL, and TI-82

Development/Procedures:

The teacher will arrange the students in groups prior to the beginning of the lesson. These groups should be no larger than four students to allow maximum utilization of the CBL. (Note: The number of available CBL's will dictate group size; this lab can be done via the teacher and one CBL.) Prior to doing this lab, students should have all necessary programs linked into their calculators.

Activity:

One student will record his heart rate via the equipment. This will produce a sinusoidal curve (see *Figure 1*). The students will use a program that will allow them to remove extraneous material from the beginning or the end of their graph. The student will follow the worksheet guidelines to approximate a sinusoidal equation. The students will compare their results with a cosine program that will generate a cosine curve (see *Figure 2*) and a cosine equation (see *Figure 3*). Discussion of data and extensions will follow.

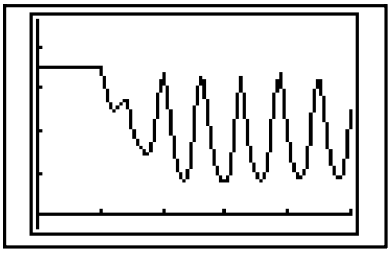

Figure 1

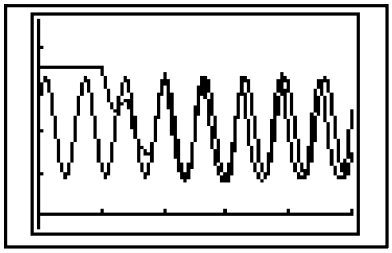

Figure 2

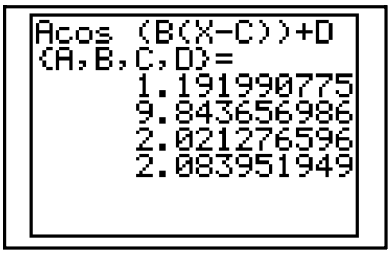

Figure 3

Evaluation:

The teacher will circulate among the groups to ensure that they are on task and to assist with any technical difficulties. Group evaluations will be based on performance: time on task, quality of discussions, and worksheet completion. A variety of extensions have been included on the worksheet. Additional extensions can be generated by the class and interaction with the medical community. Students might contact hospitals, pace maker companies, or NIH on the Internet.

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Heart Rate Worksheet

1. Connect heart monitor to the CBL-DIN.
2. Connect DIN to CBL into Channel 1 (CH1).
3. Connect CBL to TI-82 with link.
4. Turn TI-82 on and find program HEART (DO NOT ENTER).
5. Place heart monitor probe on earlobe (Remove any earrings.).
6. Turn on CBL (red button). The screen will appear with a value; this is not important.
7. Press **enter** on your TI-82.
8. Enter HEART program (Note: CBL monitor should state ready, and three bars must appear.)
9. Press **Trigger** on CBL. (It is receiving samples and will state when done.)
10. The probe can be removed from your ear while the TI-82 is graphing the data.
11. State the heart beat rate.

_____

12. Describe the curve.

_____

_____

13. Why would a heartbeat graph resemble a sinusoidal curve?

_____

_____

_____

14. Why would it not resemble a sinusoidal curve?

_____

_____

_____

15. Cleaning Graph

As discussed in question 14, some data may not be as representative of the curve as one would like. In order to remove extraneous data (at the start or end only), use a program that will select the section to be examine.

a. Press **Program**
b. **Enter** Program ALLSELCT
c. **Enter**
d. **Enter**
e. Your x values are located in L_2. (enter **2**)
f. Your y values are located in L_4. (enter **4**)
g. Move the cursor to where the curve will begin. Since a cosine equation and curve will be created, start at a maximum. Press **enter**.
h. Move the cursor to where the curve will end. Try to include as many curves as possible and end on a maximum. Press **enter**.
I. The program is now analyzing the data.
j. When it is finished, the program will prompt one to press **enter**.
k. View the "nice" graph.

16. Finding the equation of $Y = A \cos (B (x - C) + D)$

a. State the first maximum coordinate. _____
Note: *x* is the phase shift (C value)

b. State the first minimum coordinate. _____

c. Determine the amplitude, using the formula.
A = (Max - Min)/2
(Which coordinate determines height) _____

d. Determine the vertical shift (or base line).
D = (Max + Min)/2 _____

e. Determine the period, using the formula ($B = 2\pi$/answer).
Find the distance from the first maximum to the second maximum. (Which coordinate determines distance, and remember distance is always positive.) _____

State

A _____ B _____

C _____ D _____

17. Equation and Graph Generated by TI-82

    a. Press **program**
    b. **Enter** ACOS
    c. **Enter**
    d. You will be asked to move the cursor to a maximum point (then **enter**) and then to a minimum point (then **enter**). The screen will contain the calculator's guess of the equation.
    e. **Enter**. The computer regraphs the original data and then graphs the equation. (Note: If you want to see this again, press **y=**. Place cursor on the = of y_1 and press **enter**. Press **graph** (original data appears), press **y=**, and place cursor on the = y_1 and **enter**, press **graph**.) (Note: To recall the calculator's equation, press **2nd quit**.)

18. Do your equations match? _____

19. Are the graphs similar? _____

20. Suggest reasons why the actual data and the generated information may not agree. (Refer back to question 14.)

_____

_____

_____

_____

Optional Activities

21. How would exercising effect the heart rate graph and equation? (If time permits, repeat experiment.)

_____

_____

_____

22. Is there a noticeable difference in graphs and equations between
    a. male and female.
    b. young and old.
    c. awake and asleep.

    (Note: Option **a** can be done in class but options **b** & **c** may require equipment check out and an out-of-class assignment.)

23. Consult with a doctor about what an EKG looks like.

24. What changes in your heart rate might occur when additional adrenalin is introduced into the system? (i.e. fear, terror,…)

25. What other biological systems can be represented by sinusoidal curves? (Open-ended discussion)

TI-82 PROGRAMS

Program HEART

```
ClrList L1,L2,L3,L4,L5,L6
PlotsOff
Func
FnOff
AxesOn
ClrHome
{1,0}→L1
Send(L1)
{1,1,14}→L1
Send(L1)
95/5→B
ClrHome
Disp "HIT TRIGGER TO"
Disp "START MONITORING"
Disp "HEART RATE."
{3,B^-1,99,1,1,0,10,0,1}→L1
Send(L1)
ClrDraw
Get(L4)
Get(L2)
ClrList L1
1→C
For(N,2,98,1)
If L4(N-1)<L4(N) and L4(N+1)<L4(N)
Then
L2(N)→L1(C)
C+1→C
End
End
dim L1-1→dim L3
For(M,2,dim L1,1)
L1(M)-L1(M-1)→L3(M-1)
End
(sum L3-min(L3)-max(L3))/(dim L3-2)→Q
60/Q→Q
0→Xmin
5→Xmax
1→Xscl
.4*(max(L4)-min(L4))→A
min(L4)-A→Ymin
max(L4)+A→Ymax
1→Yscl
Plot1(xyLine,L2,L4,. . .)
```

```
DispGraph
Text(4,16,“RATE: ”,Q,“  BPM”):Text(51,80,“T(S)”)
Stop
```

Program ACOS

```
ClrHome
Disp “ENTER”
Disp “TRACE TO MAX”
Disp “ENTER”
Disp “TRACE TO MIN”
Disp “ENTER”
Pause
Input
X→C:Y→J
Input
X→S:Y→T
π/(S-C)→B
(T+J)/2→D
J-D→A
ClrHome
Disp “Y=”
Disp “Acos (B(X-C))+D”
Disp “{A,B,C,D}=”
Disp A,B,C,D
Pause
“Acos (B(X-C))+D”→Y1
DispGraph
```

Program ALLSELCT

```
ClrHome
Disp “YOU WILL NEED”
Disp “TO KNOW THE LIST”
Disp “UNDER WHICH THE”
Disp “X-LIST AND”
Disp “Y-LIST ARE”
Disp “STORED TO”
Disp “CONTINUE”
Pause
prgmSORTLIST
ClrHome
ClrDraw
FnOff
PlotsOff
```

```
AxesOn
Plot1(xyLine,L1,L2,…)
ZoomStat
Text(2,2,“LOWER BOUND?”)
Trace
X→A
Vertical A
Text(2,2,“UPPER BOUND?    “)
Trace
X→B
Vertical B
dim L1→N
1→C
Text(2,2,“ANALYZING…    “)
For(I,1,N,1)
If L1(I)≥A and L1(I)≤B
Then
L1(I)→L3(C)
L2(I)→L4(C)
C+1→C
End
End
ClrHome
Disp “X-LIST: L3”
Disp “Y-LIST: L4”
Disp “”
Disp “HIT ENTER TO SEE”
Disp “SELECTED PLOT.”
Pause
Plot1(xyLine,L3,L4,…)
ZoomStat
Stop
```

Program SORTLIST

```
ClrHome
Menu(“WHERE IS X-LIST?”,“L1”,1,“L2”,2,“L3”,3,“L4”,4,“L5”,5,“L6”,6)
Lbl 1:1→Q:Goto 0
Lbl 2:2→Q:Goto 0
Lbl 3:3→Q:Goto 0
Lbl 4:4→Q:Goto 0
Lbl 5:5→Q:Goto 0
Lbl 6:6→Q:Goto 0
Lbl 0
```

```
ClrHome
Menu("WHERE IS Y-LIST?","L1",A,"L2",B,"L3",C,"L4",D,"L5",E,"L6",F)
Lbl A:1→R:Goto G
Lbl B:2→R:Goto G
Lbl C:3→R:Goto G
Lbl D:4→R:Goto G
Lbl E:5→R:Goto G
Lbl F:6→R:Goto G
Lbl G
If R=1 and Q≠
3
Then:L1→L3:Goto H:End
If R=1 and Q=3
Then:L1→L2:Goto H:End
Lbl H
If Q=2
Then:L2→L1:Goto J:End
If Q=3
Then:L3→L1:Goto J:End
If Q=4
Then:L4→L1:Goto J:End
If Q=5
Then:L5→L1:Goto J:End
If Q=6
Then:L6→L1:Goto J:End
Lbl J
If R=1 and Q≠3
Then:L3→L2:Goto K:End
If R=3
Then:L3→L2:Goto K:End
If R=4
Then:L4→L2:Goto K:End
If R=5
Then:L5→L2:Goto K:End
If R=6
Then:L6→L2:Goto K:End
Lbl K
ClrList L3,L4,L5,L6
Return
Stop
```